MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

PCL0016 CALCULUS

(Foundation Engineering)

4 March 2019
2.30 p.m. – 4.30 a.m.
(2 Hours)

INSTRUCTION TO STUDENT

1. This examination paper consists of **4 pages** (including the cover page) with **6 questions only**.

2. Questions **1** and **6** are each allocated with **20** marks while Questions **2**, **3**, **4** and **5** are each allocated with **15** marks. **Attempt ALL questions.**

3. Please write all your answers in the Answer Booklet provided. **Show all relevant steps** to obtain maximum marks.

Question 1 [20 marks]

(a) Using Squeeze Theorem, compute the following limit. [5 marks]

$$\lim_{x\to\infty} x^2 e^{\sin\left(\frac{x}{2}\right)}$$

(b) Evaluate the following limits, if they exist

(i) $\lim_{x\to 2}\dfrac{x^2-x-2}{x^2-3x+2}$ [2 marks]

(ii) $\lim_{x\to\infty}\dfrac{7x^{13}+4x^2-2x+3}{3x^{16}-4x^{10}+6x^4-2x^2+7}$ [2 marks]

(iii) $\lim_{x\to 4}\dfrac{\sqrt{20-x}-x}{\sqrt{12+x}-4}$ [5 marks]

(iv) $\lim_{x\to 0}\dfrac{\sin 3x}{\tan 2x}$ [3 marks]

(c) For what value of k is the following function continuous at $x = 1$? [3 marks]

$$f(x)=\begin{cases}\dfrac{x^2-1}{x-1}, & x\neq 1\\ k, & x=1\end{cases}$$

Question 2 [15 marks]

(a) Using L'Hôpital's rule, evaluate $\lim_{x\to 0^+}\dfrac{\ln x}{\cot x}$. [3 marks]

(b) Find a and b such that the line $y=-3x+5$ is tangent to the graph $y=ax^3+bx+1$ at $x = 1$. [4 marks]

(c) Evaluate $\dfrac{dy}{dx}$ for the following functions:

(i) $e^{2x}=x^2-\ln(xy)$ [4 marks]

(ii) $y=\dfrac{\sqrt{x^3+4x^2}}{\left(x^2+x\right)^5}$ [4 marks]

Continued...

Question 3 [15 marks]

(a) A piece of wire with a length of 2 meters is cut into two pieces and one piece is bent into a square and the other is bent into an equilateral triangle, as shown in Figure 3.1. Find the values of x so that the total area enclosed by both is minimum and maximum. [8 marks]

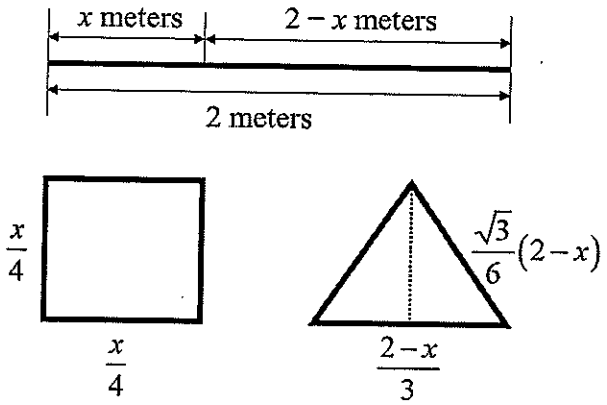

Figure 3.1

(b) A function $f(x)$ is denoted by $f(x) = x^4 - 4x^3 + 25$.

(i) Find the intervals over which f is increasing and the intervals over which f is decreasing. [3 marks]

(ii) Find the intervals over which the graph of f is concave up and over which it is concave down. [3 marks]

(iii) Identify the extrema of f. [1 mark]

Question 4 [15 marks]

(a) Given $f(x) = \dfrac{x-2}{x^3 - x^2}$. Evaluate $\int f(x)dx$ using partial fraction decomposition. [6 marks]

(b) Evaluate the following integrals:

(i) $\int x^2 \left(6 - 2x^3\right)^5 dx$ (Hint: Use integration by subsititution) [3 marks]

(ii) $\int \sin x \ln(\cos x)\, dx$ (Hint: Use integration by parts) [3 marks]

(iii) $\displaystyle\int_1^2 \frac{4x^{10} - 2x^4 + 15x^3}{x^5} dx$ [3 marks]

Continued...

Question 5 [15 marks]

(a) Find the area bounded above by $f(x)=-x^2+4x+1$ and below by $g(x)=-x^3+7x^2-10x+3$ over the interval $1 \le x \le 2$. [4 marks]

(b) A region is bounded by $f(x)=1-x^2$, $x=0$, $x=1$ and x-axis.

(i) Find the volume of the solid if the region is rotated around the x-axis. [3 marks]

(ii) Find the volume of the solid if the region is rotated around $y=-1$. [4 marks]

(c) Find the arc length of $f(x)=1+4x^{\frac{3}{2}}$ from (1, 5) to (4, 32). [4 marks]

Question 6 [20 marks]

(a) Form a differential equation for the following functions by eliminating arbitrary constant(s):

(i) $y=5ax+x^2$ [4 marks]

(ii) $ax^2+by=1$ [6 marks]

(b) Solve the following differential equations:

(i) $2yy'-3x^2=0,\ y(0)=0$ [4 marks]

(ii) $3y''+2y'-8y=0,\ y(0)=-5,\ y'(0)=20$ [6 marks]

End of Paper